S.No. 8047 **24DPMA02**

(For the candidates admitted from 2024–25 onwards)

M.Sc. DEGREE EXAMINATION, AUGUST 2025

First Semester

Maths

REAL ANALYSIS – I

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define bounded variation on [a, b].
2. Define absolute convergence.
3. Define Riemann - Stieltjes sum.
4. State the comparison theorems.
5. State the second fundamental theorem of integral calculus.
6. State the theorem on change of variable in a Riemann integral.
7. Define a double sequence.
8. State the Cesaro sum.
9. State the Cauchy condition for uniform Convergence of series.
10. Define bounded convergence.

PART B — ($3 \times 5 = 15$ marks)

Answer any THREE questions out of Five questions.

11. If f is monotonic on $[a, b]$ then prove that the set of discontinuities of f is countable.
12. If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on $[a, b]$ then prove that $c_1 f + c_2 g \in R(\alpha)$ on $[a, b]$ and we have $\int_a^b (c_1 f + c_2 g)\, d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$.
13. Let α be of bounded variation on $[a, b]$ and assume that $f \in R(\alpha)$ on $[a, b]$ then prove that $f \in R(\alpha)$ on every subinterval $[c, d]$ of $[a, b]$.

14. If each $a_n > 0$ then prove that the product $\prod(1+a_n)$ converges if and only if the series $\sum a_n$ converges.

15. Assume that f_n converges to uniformly on S. If each f_n is continuous at a point c of S then prove that the limit function f is also continuous at c.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions.

16. (a) State and prove additive property of total variation.

Or

(b) Let f be of bounded variation on $[a,b]$. If $x \in (a,b]$ let $v(x) = v_f(a,x)$ and put $v(a) = 0$. Then prove the necessary and sufficient part for every point of continuity of 'f' is also a point of continuity of v.

17. (a) State and prove the formula for integration by parts.

Or

(b) If $f \in R(\alpha)$ on $[a, b]$ then prove that $f^2 \in R(\alpha)$ on $[a, b]$.

18. (a) State and prove the second Mean–value theorem for Riemann-Stieltjes integral.

Or

(b) Let $v(x)$ denote the total variation of α on $[a, x]$ if a $< x \leq b$ and let $V(a) = 0$. Let f be defined and bounded on $[a,b]$. If $f \in R(\alpha)$ on $[a, b]$ then prove that $f \in R(v)$ on $[a, b]$.

19. (a) State and prove the Cauchy's condition for products.

Or

(b) State and prove the Abel's limit theorem.

20. (a) State and prove the Dirichlet's test for uniform convergence.

Or

(b) Assume that $f_n \to f$ uniformly on $[a, b]$ and define $g_n(x) = \int_a^x f_n(t)\,d\alpha(t)$ if $x \in [a,b]$, $n = 1, 2 \cdots$ then prove the following.

(i) $f \in R(\alpha)$ on $[a, b]$.

(ii) $g_n \to g$ uniformly on $[a, b]$ where.

$$g(x) = \int_a^x f(t)\,d\alpha(t).$$

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